

# Part A

(Please answer BOTH questions from this part.)

1.

Let  $X \subseteq \mathbb{R}$  be a finite set of monetary outcomes. Suppose the DM has some preferences  $\succsim$  over monetary lotteries  $\Delta(X)$  that admit the following utility representation:

$$W(p) = \sum_{k=1}^K p(G(x_k)) (x_k - x_{k-1});$$

where outcomes in the support of lottery  $p$  are ranked by  $x_0 < x_1 < \dots < x_K$ ,  $G(x_k) = \sum_{j=k}^K p(x_j)$ , and  $f: [0;1] \rightarrow [0;1]$  is a continuous and increasing function.

Intuitively, outcomes in the support of the lottery are ranked from the lowest to the highest. The DM counts every increment  $(x_k - x_{k-1})$  by some weight. The weight is determined by first assessing  $(G(x_k))$  the probability of the outcome of the lottery being no worse than  $x_k$ , and then "reweigh" this probability by an increasing transformation  $(f)$ .

It might be useful to know that  $W$  generalizes the standard expectation functional  $E(p) = \sum_{k=1}^K p(x_k) x_k = \sum_{k=1}^K G(x_k) (x_k - x_{k-1})$ :

Either prove or provide a counterexample to the following TRUE OR FALSE statements:

- $\succsim$  is Independent.
- $\succsim$  is Archimedean.
- $\succsim$  is Monotone: If  $p$  first order stochastically dominates  $q$ , then  $p \succsim q$  for all  $p, q \in \Delta(X)$ .

2.

Suppose there are two consumers, two states, and a single consumption good. Both consumers have expected utility functions as follows

$$\begin{aligned} U_1(x_{11}; x_{21}) &= p_{11} u_1(x_{11}) + p_{21} u_1(x_{21}); \\ U_2(x_{12}; x_{22}) &= p_{12} u_2(x_{12}) + p_{22} u_2(x_{22}); \end{aligned}$$

where  $x_{si}$  is  $i$ 's consumption in state  $s$  and  $p_{si}$  is the subjective probability of state  $s$  by consumer  $i$ . Assume further that each  $u_i$  is concave and differentiable. In particular, suppose consumer 1 is risk neutral and consumer 2 is strictly risk averse. Let  $I = (I_1; I_2)$  be the aggregate endowment of contingent commodities for these two states. Assume each consumer is endowed with half of the total resources, that is,  $(I_{1i}; I_{2i}) = \frac{1}{2} I$ .

- Suppose the two consumers have the same subjective probability so  $p_{s1} = p_{s2}$  for all  $s$ . Prove that consumer 2 will fully insure his consumption at any interior Arrow-Debreu equilibrium. Provide an intuition for why the equilibrium allocation of risk is efficient.

(b). Now suppose the two consumers have different subjective probabilities about the states so  $\pi_{s1} \neq \pi_{s2}$  for some  $s$ . Will consumer 2 fully insure at an interior Arrow-

## Part B

(Please answer TWO of the three questions from this part.)

1. Consider the following job market signaling model: A worker knows his talent but his potential employer does not. The worker's talent takes two possible values

$$= \begin{matrix} L & \text{with probability } 1 \\ H & \text{with probability } \end{matrix}$$

where  $H > L > 0$ : The value of the worker to the employer is  $E(\cdot)$ ; the expected value of  $\cdot$  given the information the employer has about  $\cdot$ . We assume that the employer pays the worker a wage  $w$  that is equal to this expectation. (Implicitly, we are assuming that the job market is competitive.)

The worker can acquire education (which the employer observes) to signal his talent. However, acquiring education is costly. The cost of obtaining

- (b) Find the correlated equilibrium of this game that generates the *lowest sum of payoffs* to the players.
3. Consider a random proposer version of the Rubinstein bargaining game played between Ann and Bob with an exogenous risk of *breakdown*. In each period Ann is selected as the proposer with probability  $\frac{1}{3}$  and Bob with probability  $\frac{1}{2}$ . With probability  $\frac{1}{6}$  the game ends with both players getting 0. The proposer (if and when there is one) makes an offer  $(x_A; x_B)$  such that  $x_A + x_B = 1$ . The responder may then accept the offer or reject it. Accepting the offer ends the game and Ann and Bob receive  $x_A$  and  $x_B$  respectively, in that period. Rejecting the offer leads the bargaining to continue to the next period. Ann and Bob have the same discount factor,